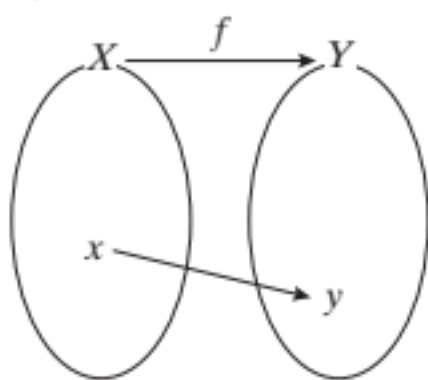


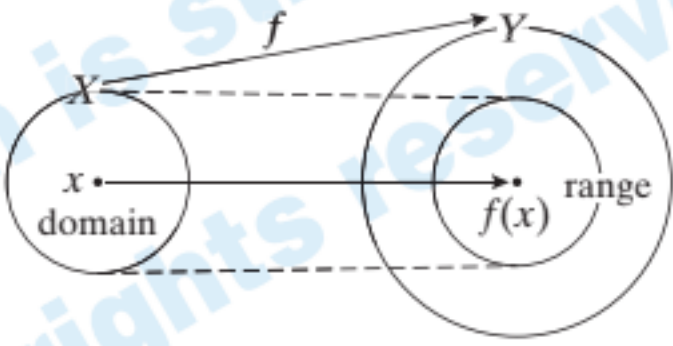
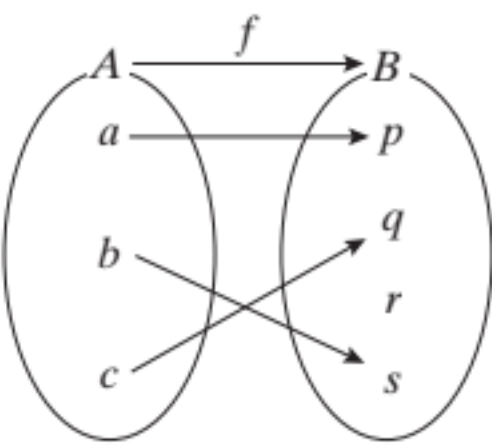
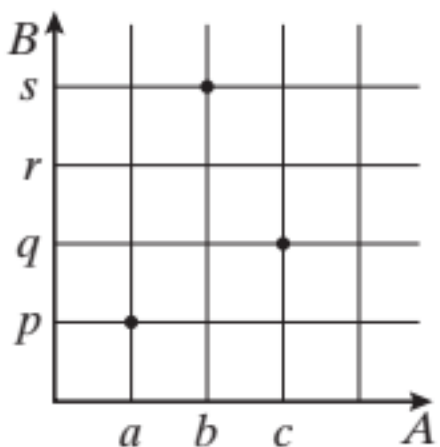
LEVEL X, SECTION XM

Topic	Formulas & Notes	Reference
Matrix Definitions, Addition, and Subtraction	When referring to a specific matrix, the order of describing the matrix <i>form</i> is: (# of rows) $\times$ (# of columns) $\begin{matrix} \text{-----} \rightarrow \\ \text{rows} \end{matrix}$ $\begin{matrix} & & & \text{columns} \\ & & \downarrow & \downarrow & \downarrow & \downarrow \\ \begin{bmatrix} 3 & 5 & 8 & 7 \\ 2 & -3 & 7 & 4 \end{bmatrix} \end{matrix}$	XM1a
	If two matrices A and B are of the same form, and the corresponding components of each matrix are equal, then A and B are equal, and this relationship may be expressed as $A = B$. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \Leftrightarrow \begin{matrix} a = p, & b = q \\ c = r, & d = s \end{matrix}$	XM4b
<i>Matrix Sum and Difference</i>	Given two matrices A and B of the same form, the sum $A + B$ and the difference $A - B$ are defined as follows. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$, then: $A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \end{bmatrix}$ $A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} & a_{13} - b_{13} \\ a_{21} - b_{21} & a_{22} - b_{22} & a_{23} - b_{23} \end{bmatrix}$	XM7a
<i>Zero Matrix</i>	A matrix in which all the components are zero is called a <i>zero matrix</i> and is designated by the letter O .	XM8a
<i>The Product of a Matrix and a Real Number</i>	Given a matrix A, and a real number k, If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, then $kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{bmatrix}$	XM9a
Matrix Multiplication	Given that $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$ Given an $m \times l$ matrix and a $p \times q$ matrix, if $l = p$, then the product of the two matrices is an $m \times q$ matrix.	XM15a
<i>Matrix Multiplication Properties</i>	$k(AB) = (kA)B = A(kB)$ (where k is a real number) $(AB)C = A(BC)$ Associative Property $A(B + C) = AB + AC$ $(A + B)C = AC + BC$ Distributive Property	XM17a
<i>Unit Matrix</i>	$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called a unit matrix. $AE = EA = A$	XM18b

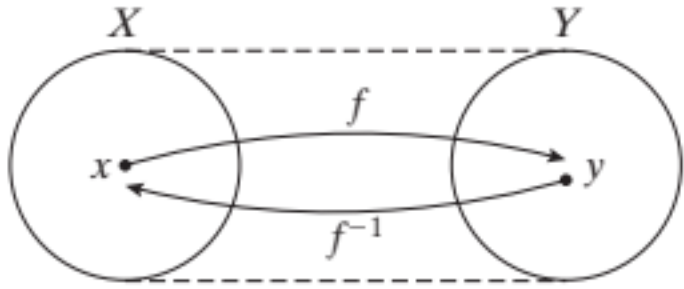
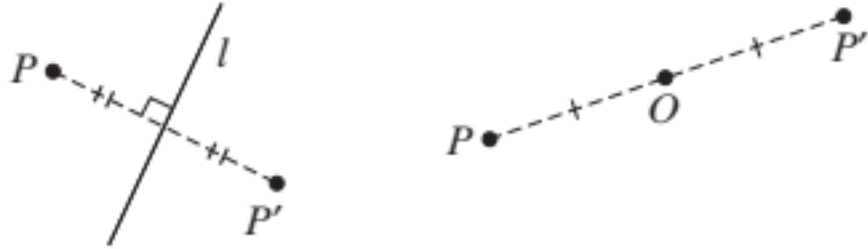
LEVEL X, SECTION XM

Topic	Formulas & Notes	Reference
Inverse Matrices <i>Zero Divisor</i>	When we multiply numbers, if $ab = 0$, then $a = 0$ or $b = 0$. However, when we multiply matrices, if $AB = O$, then $A = O$ or $B = O$ is not always true. Even if $A \neq O$ and $B \neq O$, there may be a case where $AB = O$. In such a case, A and B are both called <i>zero divisors</i>.	XM21a
<i>Inverse Matrix</i>	Given that A is a 2×2 matrix, and E is a unit matrix, if there exists a matrix B such that $AB = BA = E$, then B is called the <i>inverse matrix</i> of A, and it can be expressed as A^{-1}.	XM22a
<i>Determinant</i>	Given that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$: The quantity $ad - bc$ is called the “<i>determinant of matrix A</i>”, abbreviated as “$\det A$”, where $\det A = ad - bc$. (a) If $\det A \neq 0$, then $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. (b) If $\det A = 0$, then A^{-1} does not exist; A does not have an inverse.	XM24a
Equations and Matrices	Given the simultaneous linear equations: $\begin{cases} ax + by = p \\ cx + dy = q \end{cases}$ Letting $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} p \\ q \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $AX = B$ When $\det A = ad - bc \neq 0$, $X = A^{-1}B$	XM33a
Mapping 1	Given that the elements x of the set X correspond to the elements y of the set Y, The correspondence between x and y is known as the <i>Mapping from X to Y</i>, and can be denoted by letters, such as f, g, etc. Expressing f as a mapping from X to Y, $f: X \rightarrow Y$ A function is a mapping from one set of numbers to another set of numbers. 	XM41a

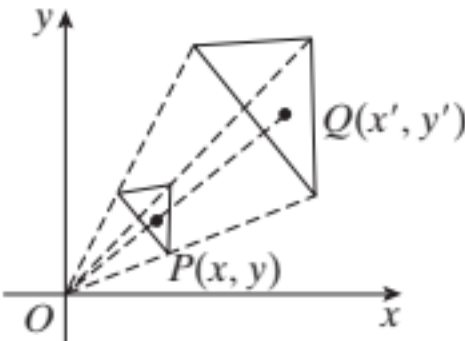
LEVEL X, SECTION XM

Topic	Formulas & Notes	Reference
	Given $f: X \rightarrow Y$, When the element x of X corresponds to the element y of Y, y is called the <i>image</i> of x through the mapping of f and is expressed as: $y = f(x)$	XM41b
Mapping	In general, given two sets, X and Y , in order for mapping from X to Y to occur, each element in X must be mapped to one and only one element in Y .	XM42a
Onto Mapping Into Mapping	Given two sets, X and Y, and the mapping $f: X \rightarrow Y$, if each element of Y is an image of at least one element of X, i.e. when $f(X) = Y$, the mapping is called an <i>onto mapping</i>. Other general mappings are called <i>into mappings</i>.	XM42b
One-to-one Mapping	Given two sets, X and Y , and the mapping $f: X \rightarrow Y$, if each element of Y is an image of at most one element of X , the mapping is called a <i>one-to-one mapping</i> .	XM43a
	Given the mapping $f: X \rightarrow Y$, The <i>domain</i> of f is set X. The <i>range</i> of f is the set of all images of the elements x in X. 	XM44a
	Given the mapping of $f: A \rightarrow B$, The points (a, p), (b, s), and (c, q) are <i>ordered pairs</i>, which can be plotted on a graph. Using a diagram of the <i>direct product</i> $A \times B$, The graph of the mapping of $f: A \rightarrow B$ is shown below on the right.  	XM45a

LEVEL X, SECTION XM

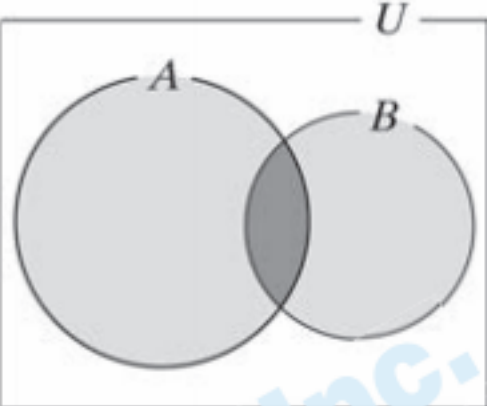
Topic	Formulas & Notes	Reference
	In general, given the onto mapping $f: X \rightarrow Y$, if the mapping is a one-to-one mapping, then the mapping has an inverse. The inverse is also a mapping, and is expressed by f^{-1}.  Note: $(f^{-1})^{-1} = f$	XM46a
Composite Mapping	Given three sets, X, Y, and Z, where f is the mapping from X to Y and g is the mapping from Y to Z, If element x in X corresponds to element y in Y, which corresponds to element z in Z, The correspondence of x to z is the mapping from X to Z. This is called the <i>Composite Mapping</i> of f and g, and is expressed by $g \circ f$.	XM48a
	For mapping, generally $f \circ g \neq g \circ f$, but $(f \circ g) \circ h = f \circ (g \circ h)$.	XM48b
Mapping 2 Transformation	The set of all points (x, y) on the coordinate plane is expressed as $R \times R$, or R^2. A <i>transformation</i> on the coordinate plane is the mapping $f: R^2 \rightarrow R^2$.	XM55a
Parallel Translation	In R^2, when the point (p, q) moves to the point $(p + a, q + b)$, the transformation can be expressed as $f: (x, y) \rightarrow (x + a, y + b)$. This is called a <i>parallel translation</i>.	XM55b
Symmetric Translation	A <i>symmetric translation</i> is a mapping obtained by translating a point with respect to another point or line. 	XM57a
Transformations 1	Given that $f: (x, y) \rightarrow (x', y')$, the mapping f can be expressed by: $\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases} \rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ The mapping f is called a <i>Linear Transformation</i>, where $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is known as the matrix of the linear transformation f.	XM61a

LEVEL X, SECTION XM

Topic	Formulas & Notes	Reference
<i>Similar Transformation</i>	A <i>Similar Transformation</i> is the mapping by which $P(x, y)$ is mapped to point $Q(x', y') = (kx, ky)$, where the center is the origin. k is known as the ratio of similitude. 	XM62b
	In a linear transformation, $f: (0, 0) \rightarrow (0, 0)$. If $f: (0, 0) \nrightarrow (0, 0)$, The mapping is not a linear transformation.	XM64a
Transformations 2	Given that A is the matrix of the linear transformation f and the inverse matrix A^{-1} of A exists, then the inverse mapping of f is also a linear transformation and its matrix is A^{-1}.	XM71a
	Given that a region is transformed by the matrix of transformation $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the area of the region is increased by a factor of $ad - bc$ (where $ad - bc \neq 0$).	XM79a
Transformations 3	If the matrices of the linear transformations f and g are A and B, respectively, then the matrix of the composite transformation $g \circ f$ is BA. Similarly, the matrix of the composite transformation $f \circ g$ is AB.	XM81b
	Given that $\vec{v}' = f(\vec{v})$, vector $\vec{v}'$ is the image of vector $\vec{v}$ by the linear transformation f.	XM84b
<i>Linearity of Linear Transformations</i>	$f(\vec{p}_1 + \vec{p}_2) = f(\vec{p}_1) + f(\vec{p}_2)$ $f(k\vec{p}) = kf(\vec{p})$ $f(k\vec{p}_1 + l\vec{p}_2) = kf(\vec{p}_1) + lf(\vec{p}_2)$	XM85a

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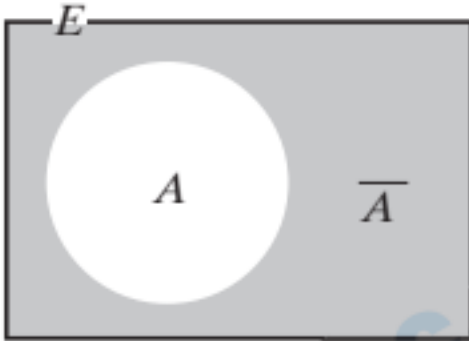
LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
Introduction to Permutations	A <i>set</i> is a collection of objects that satisfy a certain condition, and <i>elements</i> are the individual objects that compose a set. Given two sets A and B that share some common elements, $n(A)$ represents the number of elements in A, and $n(B)$ represents the number of elements in B. - The <i>intersection</i> of A and B, denoted as $A \cap B$, is the set of common elements. (darker shading) - The <i>union</i> of A and B, denoted as $A \cup B$, is the set of elements that are contained in either A or B. (lighter and darker shading) 	XP1b
	Given two sets, A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$	XP2a
	Given the two sets, $A = \{a_1, a_2, a_3, \dots\} \text{ and } B = \{b_1, b_2, b_3, \dots\},$ the <i>product</i> of A and B is a new set denoted as $A \times B$, and its elements are: $(a_1, b_1), (a_1, b_2), \dots, (a_2, b_1), (a_2, b_2), \dots$	XP3a
	Given two sets, A and B, $n(A \times B) = n(A) \times n(B)$	XP3b
Permutations	The number of permutations possible out of n objects, where r objects are used at a time is denoted by ${}_nP_r$. ${}_nP_r = n(n-1)(n-2)(n-3)\dots(n-r+1)$ $(n \geq r, \quad n, r > 0)$ ${}_nP_n = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1$	XP11a
	$n! = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1 \quad (\text{where } n \text{ is a natural number})$	XP11b
	${}_nP_r = \frac{n!}{(n-r)!}$ $(n \geq r)$ ${}_nP_n = n!$ $0!$ is defined as 1	XP12a
Permutations and Combinations	The number of permutations possible out of n objects, (where p of one kind are alike, q of another kind are alike, $\dots$ and r of yet another kind are alike) can be expressed as: $\frac{n!}{p! \, q! \, \dots \, r!} \quad (\text{where } p + q + \dots + r = n)$	XP21b

LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
<i>Repeated Permutations</i>	The number of repeated permutations in a set of n objects where r objects are used at a time is: ${}_n\Pi_r = n^r$	XP23a
<i>Circular Permutations</i>	The ways of arranging different objects in a ring shape are called <i>circular permutations</i>. The number of such permutations of n objects is: $\frac{n!}{n} = (n - 1)!$	XP24a
	The number of permutations of n objects on a <i>reversible ring</i> is: $\frac{(n - 1)!}{2}$	XP25a
<i>Combination</i>	The number of combinations possible out of n objects, where r objects are used at a time is denoted by ${}_nC_r$. ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n(n - 1)(n - 2)\dots(n - r + 1)}{r!}$ ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n - r)!} \text{ (where } {}_nC_0 = 1, {}_nC_r = {}_nC_{n-r}, n \geq r \text{)}$	XP26b
Combinations <i>Repeated Combination</i>	A <i>repeated combination</i> where n objects are taken r at a time is denoted by ${}_nH_r$. ${}_nH_r = {}_{n+r-1}C_r = \frac{(n + r - 1)(n + r - 2)\dots(n + 2)(n + 1)n}{r!}$	XP32a
Binomial Theorem	$(a + b)^n = \sum_{r=0}^n {}_nC_r a^{n-r} b^r$ $= a^n + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 + \dots$ $\dots + {}_nC_r a^{n-r} b^r + \dots + {}_nC_{n-1} a b^{n-1} + b^n$	XP42b
	Given that the expression $(a + b + c)^n$ is expanded, the general term is: $\frac{n!}{p! q! r!} a^p b^q c^r \quad (\text{where } p + q + r = n)$	XP45b
Probability	Letting $p(A)$ be the probability of event A, $n(A)$ be the number of elementary events in A, and $n(E)$ be the number of elementary events in the whole event, $p(A) = \frac{n(A)}{n(E)}$	XP52a

LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
	$0 \leq p(A) \leq 1$ - The probability of the whole event, E, is 1. - The probability of the empty set, ϕ, is 0.	XP55a
	For any event A, the event that A does not occur is called the <i>complement event of A</i>. It is denoted as $\overline{A}$ and is read as “A bar.” Thus, $A \cup \overline{A} = E, A \cap \overline{A} = \phi$ and $p(A) + p(\overline{A}) = 1$ (where E is the whole event, and ϕ is the empty event) 	XP55b
	Given two events A and B, $p(A \cup B) = p(A) + p(B) - p(A \cap B)$	XP56a
Addition Theorem for Probabilities	If the two events A and B are mutually exclusive, then $p(A \cup B) = p(A) + p(B)$	XP59a
Conditional Probability Multiplication Theorem of Conditional Probability	The probability of events A and B both occurring is: $p(A \cap B) = p(A) \cdot p(B A)$ For three events, A, B, and C, $p(A \cap B \cap C) = p(A) \cdot p(B \cap C A) = p(A) \cdot p(B A) \cdot p(C A \cap B)$ where $(B \cap C A)$ is the conditional probability of $B \cap C$, given A, and $(C A \cap B)$ is the conditional probability of C, given $A \cap B$.	XP62a
	If events A and B are <i>independent</i>, then: $p(B A) = p(B) \quad \text{and} \quad p(A \cap B) = p(A) \cdot p(B)$ If events A and B are <i>dependent</i>, then: $p(B A) \neq p(B) \quad \text{and} \quad p(A \cap B) \neq p(A) \cdot p(B)$	XP64a
	Given events A & B, B & C, and C & A, if each pair consists of two independent events, and if any one of A, B, or C is independent to the other two, then A, B, and C are all independent. Therefore, $p(A \cap B \cap C) = p(A) \cdot p(B) \cdot p(C)$	XP65a
Independent Trials Theorem of Independent Trials	Let p be the probability of an event occurring. Given that a trial is repeated n times, the probability of the event occurring exactly r times is: ${}_nC_r \cdot p^r \cdot q^{n-r} \quad (\text{where } q = 1 - p)$	XP72a

LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
Expected value	Given that the variable X takes on one value among $x_1, x_2, \dots, x_n$, whose respective probabilities are $p_1, p_2, \dots, p_n$, $E(X) = \sum_{r=1}^n x_r p_r = x_1 p_1 + x_2 p_2 + \dots + x_n p_n$ where $E(X)$ is the <i>expected value</i>, or average value, of X.	XP82a

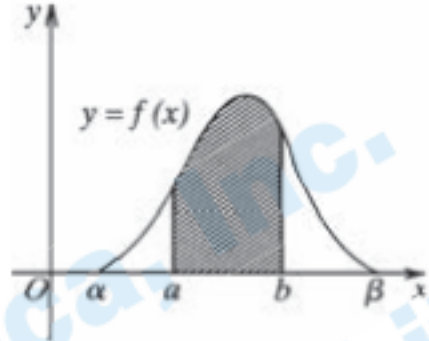
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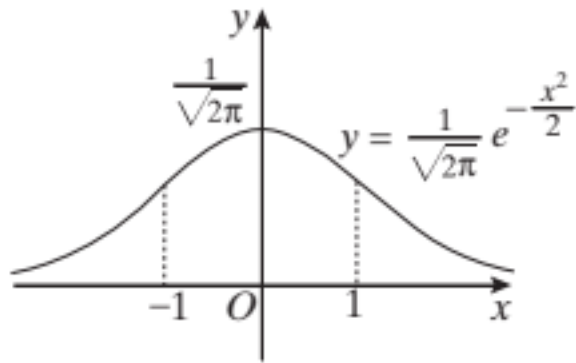
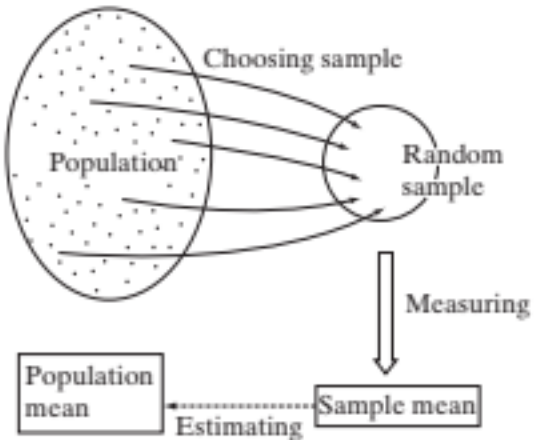
LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference														
Statistics 1	The <i>probability distribution</i> of variable X is the arrangement of all of the values of X, and their corresponding probabilities. X is called a <i>random variable</i>. The probability that X equals 2 is written as $P(X = 2)$.	XS1a														
Mean Value	Letting the probability distribution of the random variable X be: <table border="1"><tr><td>X</td><td>x_1</td><td>x_2</td><td>$\dots$</td><td>x_i</td><td>$\dots$</td><td>x_n</td></tr><tr><td>$P(X)$</td><td>p_1</td><td>p_2</td><td>$\dots$</td><td>p_i</td><td>$\dots$</td><td>p_n</td></tr></table> Here, the expected value, $E(X) = x_1p_1 + x_2p_2 + \dots + x_np_n = \sum_{i=1}^n x_ip_i$ This is also known as the <i>mean value</i> of X, denoted as μ.	X	x_1	x_2	$\dots$	x_i	$\dots$	x_n	$P(X)$	p_1	p_2	$\dots$	p_i	$\dots$	p_n	XS3a
X	x_1	x_2	$\dots$	x_i	$\dots$	x_n										
$P(X)$	p_1	p_2	$\dots$	p_i	$\dots$	p_n										
Variance	If X is a random variable and μ is the mean value of X, then the <i>variance</i> of X is: $\sigma^2 = E[(X - \mu)^2] = \sum_{i=1}^n (x_i - \mu)^2 \cdot p_i$	XS6a														
Standard Deviation	$\sigma = \sqrt{\sigma^2}$	XS6a														
Statistics 2 Binomial Distribution	From the <i>Theorem of Independent Trials</i>, If p is the probability of event E occurring in 1 trial, then the probability, P_r, of event E occurring r times in n trials is: $P_r = {}_nC_r \cdot p^r \cdot q^{n-r} \quad (\text{where } q = 1 - p)$ Letting X be the number of times event E occurs in n trials, The values of X are: $0, 1, 2, \dots, r, \dots, n$ Their respective probabilities are: $q^n, {}_nC_1 \cdot p \cdot q^{n-1}, {}_nC_2 \cdot p^2 \cdot q^{n-2}, \dots, {}_nC_r \cdot p^r \cdot q^{n-r}, \dots, p^n$ Since these terms are the same as those appearing in the expansion of the binomial theorem, which are: $(q + p)^n = q^n + {}_nC_1 \cdot p \cdot q^{n-1} + \dots + {}_nC_r \cdot p^r \cdot q^{n-r} + \dots + p^n,$ This kind of probability distribution is called a <i>Binomial Distribution</i>, and is denoted by $B(n, p)$.	XS11a														

LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference
	In the binomial distribution $B(n, p)$, the mean value is: $\mu = n \cdot p$, the variance is: $\sigma^2 = n \cdot p \cdot q$, (where $q = 1 - p$) and the standard deviation is: $\sigma = \sqrt{\sigma^2} = \sqrt{n \cdot p \cdot q}$	XS13a
	Given that $Y = aX + b$, where X and Y are random variables, $\mu_Y = a \cdot \mu_X + b, \quad \sigma_Y^2 = a^2 \cdot \sigma_X^2$	XS15a
Statistics 3	If X is a variable that has continuous real values on the frequency distribution curve $y = f(x)$, the probability that X will be a value in the interval $a \leq x \leq b$ is: $P(a \leq X \leq b) = \int_a^b f(x) dx$ (where $\int_a^\beta f(x) dx = 1$)  In this case X is called a <i>continuous random variable</i>, and $f(x)$ is called the <i>probability density function</i> of X.	XS21a
	Given that over the interval $[\alpha, \beta]$, the probability density function of X is $f(x)$, then: $\mu = \int_\alpha^\beta x \cdot f(x) dx$ $\sigma^2 = \int_\alpha^\beta [x - \mu]^2 \cdot f(x) dx$ $\sigma = \sqrt{\sigma^2}$ (where μ is the mean value of X, σ^2 is the variance of X, and σ is the standard deviation of X)	XS23a
Statistics 4 <i>Normal Distribution</i>	As n becomes large, the graph of the binomial distribution $B(n, p)$ becomes a symmetric bell curve with the mean value at the center. This curve is a <i>Normal Distribution Curve</i>, whose function is: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ (where $\int_{-\infty}^{\infty} f(x) dx = 1$, $\int_{-\infty}^{\infty} xf(x) dx = \mu$, and $\int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \sigma^2$) A distribution of the form given above for a probability density function $f(x)$ is called a <i>Normal Distribution</i> and is expressed as $N(\mu, \sigma^2)$, where μ is the mean value and σ is the standard deviation.	XS31b

LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference
Standard Normal Distribution	The <i>Standard Normal Distribution</i> is the normal distribution with mean value $\mu = 0$ and standard deviation $\sigma = 1$. It is denoted as $N(0, 1)$. The graph of the standard normal distribution is: 	XS32a
	Given that the random variable X follows the standard normal distribution $N(0, 1)$, $P(X \geq 0) = P(X \leq 0) = 0.5$ $P(X \geq a) = P(X \leq -a)$ $P(X = a) = 0 \quad \therefore P(X \geq a) = P(X > a)$ $P(X \geq a) = 1 - P(X \leq a)$ $P(X \leq a) = 2 \cdot P(0 \leq X \leq a)$ (where $a > 0$) $P(X \geq a) = 0.5 - P(0 \leq X \leq a)$ (where $a > 0$)	XS32a
	Given that the random variable X follows the normal distribution $N(\mu, \sigma^2)$, then the probability distribution of the random variable U is the standard normal distribution, where U is defined as: $U = \frac{X - \mu}{\sigma}$	XS36a
Statistics 5	Given that the random variable X follows the binomial distribution $B(n, p)$, X follows the normal distribution $N(np, npq)$ as n becomes large. Therefore, the probability distribution of the random variable U follows the standard normal distribution, where U is defined as: $U = \frac{X - \mu}{\sigma}$ (where $\mu = np$, $\sigma = \sqrt{npq}$, and $q = 1 - p$)	XS41a
Statistics 6	A <i>Sample Survey</i> is the statistical method of making estimates about the whole by examining only a randomly chosen part of it. The whole is called the <i>population</i> and the randomly chosen part is called the <i>random sample</i>. 	XS51a

LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference
	Given that a random sample of n objects is taken from a population whose distribution has a mean value of μ_X with standard deviation σ_X, The sample average, $\bar{X}$, forms a distribution whose mean value is μ_X with standard deviation $\frac{\sigma_X}{\sqrt{n}}$. As n becomes larger, the distribution of $\bar{X}$ approaches the normal distribution $N\left(\mu_X, \frac{\sigma_X^2}{n}\right)$.	XS52b
	Given a population whose standard deviation is σ and a random sample of n objects whose mean value is $\bar{X}$, The population mean value, μ, can be estimated with 95% and 99% confidence: 95% Confidence Interval: $\bar{X} - \frac{1.96\sigma}{\sqrt{n}} < \mu < \bar{X} + \frac{1.96\sigma}{\sqrt{n}}$ 99% Confidence Interval: $\bar{X} - \frac{2.58\sigma}{\sqrt{n}} < \mu < \bar{X} + \frac{2.58\sigma}{\sqrt{n}}$	XS54a
	Given a random sample of n objects, where $\bar{p}$ of the objects have a certain characteristic, The portion p of the population of size N that shares the same characteristic can be estimated with 95% and 99% confidence: 95% Confidence Interval: $\bar{p} - 1.96 \cdot \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} < p < \bar{p} + 1.96 \cdot \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$ 99% Confidence Interval: $\bar{p} - 2.58 \cdot \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} < p < \bar{p} + 2.58 \cdot \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$ (N must be large in comparison to n, and n also must be sufficiently large.)	XS58a

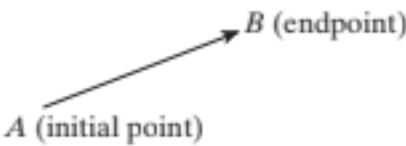
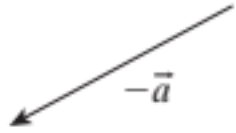
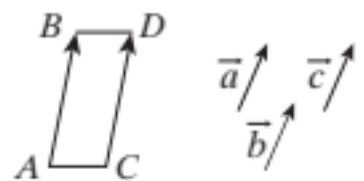
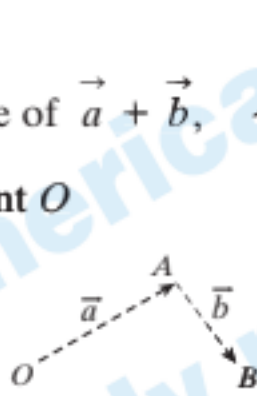
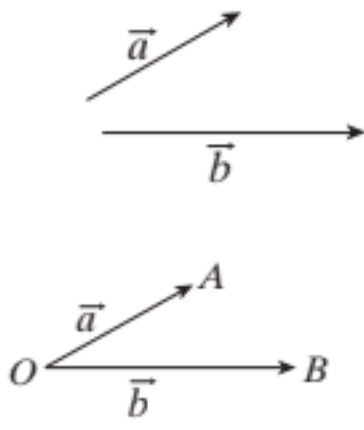
[NOTES]

LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference
Statistics 7 Hypothesis Testing	Given a random sample of n objects, whose mean is $\bar{X}$ and standard deviation is s, Letting σ be the standard deviation of the population, Test the hypothesis that the mean of the population is μ, by letting $u = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}. \text{ (If } \sigma \text{ is not given, let } \sigma = s.)$ - Using a significance of 5%, If $u > 1.96$, then the hypothesis should be rejected. If $u \leq 1.96$, then the hypothesis should not be rejected. - Using a significance of 1%, If $u > 2.58$, then the hypothesis should be rejected. If $u \leq 2.58$, then the hypothesis should not be rejected.	XS64a

[NOTES]

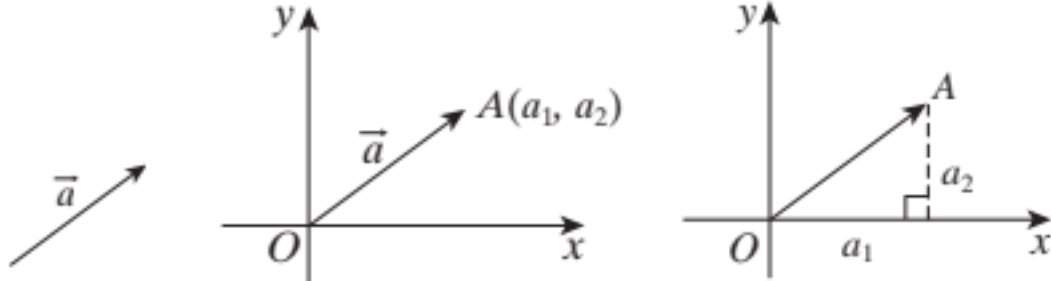
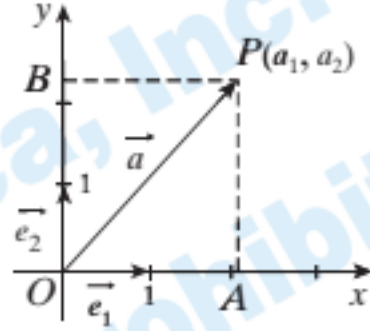
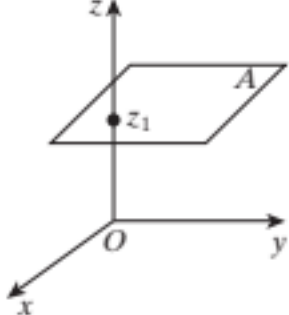
LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
Surface Vectors 1	A vector is denoted by a line segment having an <i>initial point</i> and an <i>endpoint</i>.  The vector on the right is written as: $\overrightarrow{AB}$ Its magnitude is expressed as: $\overrightarrow{AB}$ 	XV1a
	A vector of the same magnitude as $\vec{a}$ but opposite direction is written as: $-\vec{a}$  Two vectors are equal if they have the same magnitude and direction. In the figures on the right, $\overrightarrow{AB} = \overrightarrow{CD}$, and $\vec{a} = \vec{b} = \vec{c}$. 	XV1b
	Given two vectors $\vec{a}$ and $\vec{b}$, In order to determine the value of $\vec{a} + \vec{b}$, Setting $\vec{a}$ to have an initial point O and an endpoint A, and Setting $\vec{b}$ to have initial point A and an endpoint B,  Note that $\vec{b}$ is set to have its initial point at the endpoint of $\vec{a}$. Vector $\overrightarrow{OB}$ represents $\vec{a} + \vec{b}$, where $\overrightarrow{OB} = \vec{a} + \vec{b}$.  Note that $\overrightarrow{OB}$ has the same initial point as vector $\vec{a}$ and the same endpoint as vector $\vec{b}$.	XV2a
	From the figure on the right, we can express the sum of the vectors as: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ Rearranging the equation, $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$ 	XV4a
	Given two vectors $\vec{a}$ and $\vec{b}$, In order to determine the difference of the two vectors, Setting $\vec{a}$ to have an initial point O and an endpoint A, and Setting $\vec{b}$ to have initial point O and an endpoint B,  Vector $\overrightarrow{BA}$ represents $\vec{a} - \vec{b}$, where $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \vec{a} - \vec{b}$. Vector $\overrightarrow{AB}$ represents $\vec{b} - \vec{a}$, where $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$.	XV4b
	$\vec{a} - \vec{b}$ may be expressed as $\vec{a} + (-\vec{b})$.	XV5a
Zero Vector	A vector with magnitude 0 is called a <i>zero vector</i> and can be expressed as $\vec{0}$. Its direction is undefined.	XV5b

LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
Real Number Multiples of Vectors	When a non-zero vector $\vec{a}$ is multiplied by a real number k, the resulting vector $k\vec{a}$ has the following characteristics: (a) When $k > 0$, $k\vec{a}$ points in the same direction as $\vec{a}$, and its magnitude is the product of k and the length of $\vec{a}$. (b) When $k < 0$, $k\vec{a}$ points in the opposite direction as $\vec{a}$, and its magnitude is the product of k and the length of $\vec{a}$. (c) When $k = 0$, $0\vec{a} = \vec{0}$. Therefore, $k\vec{a}$ is a zero vector. Its magnitude is 0, and its direction is undefined.	XV6b
Properties of Vector Calculations	$(mn)\vec{a} = m(n\vec{a})$ $(m + n)\vec{a} = m\vec{a} + n\vec{a}$ $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$	XV7a
Parallel and Co-Linear Conditions for Vectors	(a) Parallel conditions: Given two nonzero vectors, $\vec{a}$ and $\vec{b}$, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{b} = k\vec{a}$ (where k is a real number) (b) Co-linear conditions: Point P lies on line $AB \Leftrightarrow \vec{AP} = k\vec{AB}$ (where k is a real number) 	XV8a
Position Vector	Letting O be a fixed point in a plane, the vector from O to a point P can be expressed as $\vec{OP}$. $\vec{OP}$ represents the position of point P. Therefore, $\vec{OP}$ is called the <i>position vector</i> of point P. 	XV9a
Surface Vectors 2	A point A whose position vector is $\vec{OA} = \vec{a}$, can be expressed as $A(\vec{a})$.	XV11b
	Given that a point C divides line segment AB in the ratio of $m : n$, (a) When $m > 0$ and $n > 0$, C internally divides AB in the ratio of $m : n$. (b) When $mn < 0$, C externally divides AB in the ratio of $m : n$. The position vector of point C which divides line segment AB in the ratio of $m : n$ (as described in the two cases above), is $\vec{c} (= \vec{OC})$ and is expressed as: $\vec{c} = \frac{n\vec{a} + m\vec{b}}{m + n}$	XV12a

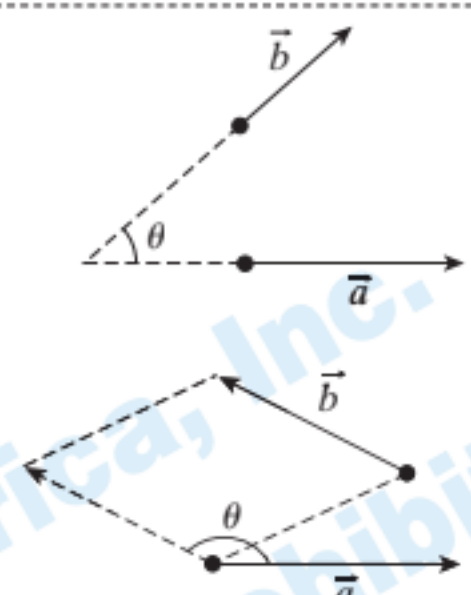
LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
Surface Vectors 3	Given that a vector $\vec{a}$ is placed in the coordinate grid as vector $\vec{OA}$, with components a_1 and a_2, we can express the vector as $\vec{a} = (a_1, a_2)$. When $\vec{a} = (a_1, a_2)$, $\vec{a} = \sqrt{a_1^2 + a_2^2}$ When $\vec{a} = 1$, $\vec{a}$ is called a <i>unit vector</i>. 	XV25a
<i>Fundamental Vectors</i>	The unit vectors on the x and y axes are $\vec{e}_1 = (1, 0)$ and $\vec{e}_2 = (0, 1)$, respectively. They are called the <i>fundamental vectors</i>. When $\vec{a} = (a_1, a_2)$, $\vec{a} = \vec{OP} = \vec{OA} + \vec{OB}$ $= a_1\vec{e}_1 + a_2\vec{e}_2$ This is called the <i>fundamental vector expression</i>. 	XV26a
<i>Operations using Components</i>	$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ $(a_1, a_2) - (b_1, b_2) = (a_1 - b_1, a_2 - b_2)$ $m(a_1, a_2) = (ma_1, ma_2)$	XV28a
Coordinates in Space <i>Parallel Translations</i>	Given the <i>parallel translation</i> of the origin $(0, 0, 0)$ to the point (α, β, γ), an arbitrary point with coordinates (x, y, z) will be moved to: $(x + \alpha, y + \beta, z + \gamma)$	XV42b
	As shown in the figure on the right, plane A is parallel to the xy-plane, and intersects the z-axis. Letting the point of intersection of plane A with the z-axis be point $(0, 0, z_1)$, the equation of plane A can be written as: $z = z_1$ In this case, the equation $z = 0$ represents the xy-plane. 	XV43b

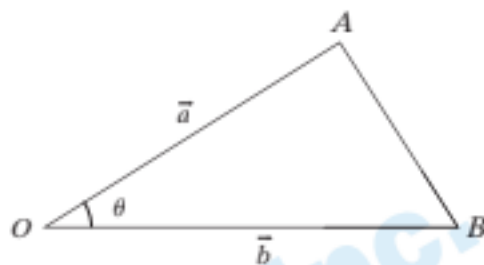
LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
<i>Internal and External Dividing Points</i>	Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the points that divide segment AB in the ratio of $m : n$ are as follows: Internally: $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$ Externally: $\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n} \right)$ (where $m \neq n$) The midpoint of segment AB can be expressed as: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	XV44a
	The distance from the origin O to a point $P(x, y, z)$ is expressed as: $OP = \sqrt{x^2 + y^2 + z^2}$	XV46b
<i>Distance between Two Points</i>	The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is expressed as: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	XV46b
<i>Equation of a Sphere</i>	The equation of a sphere with radius r, and center with coordinates (a, b, c), can be expressed as: $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$	XV49a
Vectors in Space	If we reverse the direction of $\overrightarrow{AB}$, $\overrightarrow{BA}$ is called the <i>inverse vector</i> of $\overrightarrow{AB}$, and this can be expressed as $\overrightarrow{BA} = -\overrightarrow{AB}$.	XV51a
	$\vec{a} = a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3$ (fundamental vector expression) $\vec{a} = (a, b, c)$ (components) The components of $\vec{e}_1, \vec{e}_2, \vec{e}_3$, and $\vec{0}$ are expressed as: $\vec{e}_1 = (1, 0, 0), \quad \vec{e}_2 = (0, 1, 0), \quad \vec{e}_3 = (0, 0, 1), \quad \vec{0} = (0, 0, 0)$	XV52b
<i>Operations with Components</i>	$(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$ $m(a_1, a_2, a_3) = (ma_1, ma_2, ma_3) \quad (\text{where } m \text{ is a real number})$	XV53a
	Given that $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, m \neq 0$, $\vec{a} = m\vec{b} \Leftrightarrow \vec{a} \parallel \vec{b}$	XV57a
	Given any two vectors without zero components, $\vec{a} = (a_1, a_2, a_3), \quad \vec{b} = (b_1, b_2, b_3),$ $\text{and } \vec{a} \parallel \vec{b} \Leftrightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$	XV57a

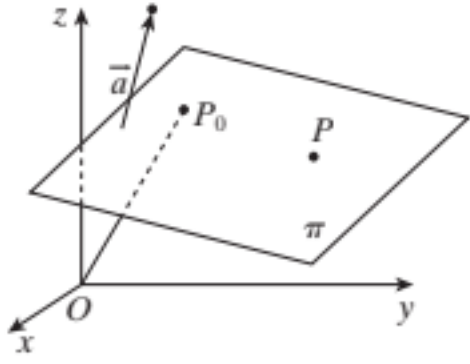
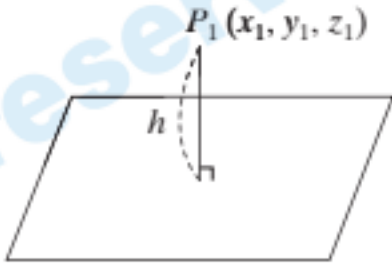
LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
Basics for Inner Products of Vectors <i>Magnitude of Vectors</i>	If $\vec{a} = (a_1, a_2, a_3)$, then $ \vec{a} = \sqrt{a_1^2 + a_2^2 + a_3^2}$	XV61a
	Given that two vectors $\vec{a}$ and $\vec{b}$ form an angle θ , (where $0^\circ \leq \theta \leq 180^\circ$), letting $\vec{c} = \vec{a} + \vec{b}$, $ \vec{c} = \sqrt{ \vec{a} ^2 + \vec{b} ^2 + 2 \vec{a} \vec{b} \cos\theta}$	XV65a
Inner Product	Given that two vectors $\vec{a}$ and $\vec{b}$ form an angle θ, where $0^\circ \leq \theta \leq 180^\circ$, $\vec{a} \vec{b} \cos\theta$ is called the <i>inner product</i> of $\vec{a}$ and $\vec{b}$. This product can be expressed as $\vec{a} \cdot \vec{b}$ or $(\vec{a}, \vec{b})$. $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos\theta$ If $\vec{a} = \vec{0}$ or if $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$ 	XV66a
Components and Inner Products of Vectors in Space	If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$, then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.	XV67b
	The following formula may be used to determine the angle between two surface vectors $\vec{a}$ and $\vec{b}$. $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} } \quad (0^\circ \leq \theta \leq 180^\circ)$	XV69a
Inner Products of Vectors	$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} } = \frac{a_1b_1 + a_2b_2}{\sqrt{a_1^2 + a_2^2} \sqrt{b_1^2 + b_2^2}} \quad (0^\circ \leq \theta \leq 180^\circ)$	XV71a
	If two vectors $\vec{a}$ and $\vec{b}$ are perpendicular, then their inner product is zero. If the inner product of two vectors $\vec{a}$ and $\vec{b}$ is zero, then the vectors are perpendicular. $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0 \quad (\text{where } \vec{a} \vec{b} \neq 0)$	XV72a
	Given two vectors $\vec{a}$ and $\vec{b}$, The sum: $\vec{a} + \vec{b}$ is a vector. The difference: $\vec{a} - \vec{b}$ is a vector. The inner product: $\vec{a} \cdot \vec{b}$ is not a vector. It is a <i>scalar</i>.	XV76a

LEVEL X, SECTION XV

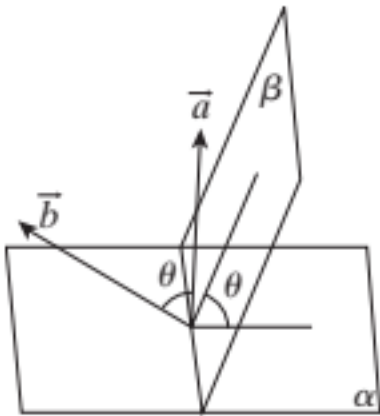
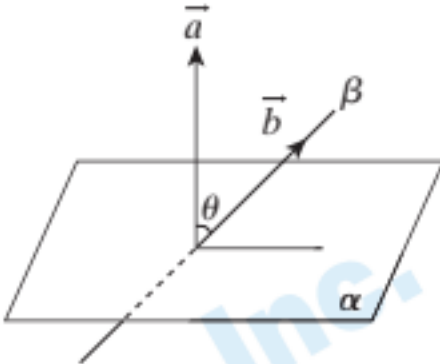
Topic	Formulas & Notes	Reference
Properties of Vectors	$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ $\vec{a} \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$ $(\vec{a} \cdot \vec{b})\vec{c} \neq \vec{a}(\vec{b} \cdot \vec{c})$ $\vec{a} \cdot \vec{b} \cdot \vec{c} \text{ has no meaning}$	XV76b
	$\vec{a} \cdot \vec{a} = \vec{a} ^2$ $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{a} + \vec{b} ^2$	XV77a
Applications and Summary of Vectors	Letting $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$ The area of $\triangle OAB$ is: $S = \frac{1}{2} \sqrt{ \vec{a} ^2 \vec{b} ^2 - (\vec{a} \cdot \vec{b})^2}$ 	XV83a
Area of a Parallelogram	$S = \sqrt{ \vec{a} ^2 \vec{b} ^2 - (\vec{a} \cdot \vec{b})^2}$	XV84a
Vectors and Figures	Given two points $A(\vec{a})$ and $B(\vec{b})$: The position vector $\vec{p}$ of point P, where P internally divides line segment AB in the ratio of $m : n$, is expressed as: $\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$ The position vector $\vec{q}$ of point Q, where Q externally divides line segment AB in the ratio of $m : n$, is expressed as: $\vec{q} = \frac{m\vec{b} - n\vec{a}}{m - n} \text{ (where } m \neq n\text{)}$	XV93a
	$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ (Note: When $\vec{a}$ or $\vec{b}$ are non-zero vectors, if $\vec{a} \cdot \vec{b} = 0$, then $\vec{a} \perp \vec{b}$.)	XV97a
Equations of Lines, Planes, and Figures 1	The equation of a line passing through a point $P_0(x_0, y_0, z_0)$ and parallel to a vector $\vec{a} = (a, b, c)$ is expressed as: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$ $\vec{a}$ is called a direction vector.	XV103a
	The equation of the line passing through two points (x_0, y_0, z_0) and (x_1, y_1, z_1) can be expressed as: $\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$	XV106b

LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
Equations of Lines, Planes, and Figures 2	In the diagram shown below, plane π passes through points P and P_0, and is perpendicular to vector $\vec{a}$. Given that $\vec{p_0}$ and $\vec{p}$ are the position vectors for P_0 and P, respectively, $\vec{P_0P} = \vec{p} - \vec{p_0}$ Since $\vec{P_0P}$ is perpendicular to $\vec{a}$, $\vec{a} \cdot (\vec{p} - \vec{p_0}) = 0$ This is known as the <i>vector equation</i> of the plane. 	XV111a
	Given that a plane passes through point $P_0(x_0, y_0, z_0)$ and is perpendicular to vector $\vec{a} = (a, b, c)$, the equation of the plane is expressed as: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$	XV111b
	Given that the distance from the plane to the origin is d, the equation of the plane is: $lx + my + nz = d \quad (\text{when } l^2 + m^2 + n^2 = 1)$ This is known as the <i>standard form</i>.	XV115b
	Given a point $P_1(x_1, y_1, z_1)$ and plane $ax + by + cz + d = 0$, the length of the perpendicular from the point to the plane is: $h = \frac{ ax_1 + by_1 + cz_1 + d }{\sqrt{a^2 + b^2 + c^2}}$ 	XV117a
Equations of Lines, Planes, and Figures 4	The angle formed by the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is the same as the angle formed by the two vectors $\vec{a} = (a_1, b_1)$ and $\vec{b} = (a_2, b_2)$, since $\vec{a}$ and $\vec{b}$ are perpendicular to the two lines respectively. Letting α be the angle formed by the two vectors, If $\alpha > 90^\circ$, the angle formed by the two lines is $(180^\circ - \alpha)$.	XV131a XV131b

[NOTES]

LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
	Given two planes α and β, The angle formed by the two planes is the same as the angle formed by the two vectors $\vec{a}$ and $\vec{b}$, where vectors $\vec{a}$ and $\vec{b}$ are the normal vectors of planes α and β, respectively. 	XV132a
	Given plane α and line β, The angle formed by the plane and the line is the complement of the angle formed by the two vectors $\vec{a}$ and $\vec{b}$, where vector $\vec{a}$ is the normal vector of plane α and vector $\vec{b}$ is the direction vector of line β. 	XV133a
	Given a curved surface S and a point $P(\vec{p})$, the <i>vector equation</i> of S is the condition for P to lie on S and is expressed as an equality in terms of the position vector, $\vec{p}$. Given the vector equation of S, to determine the equation of the surface, substitute $\vec{p} = (x, y, z)$ into the vector equation.	XV136a

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